AP Calculus AB — Homework 0219 — Name:

Dr. Paul L. Bailey — Wednesday, February 19, 2025

Write your homework *neatly*, *in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. When appropriate, *write in sentences.*

The main theory of section 4.4 is summarized below. Note that our definition for "concave up" and "concave down" is more general than that in Thomas.

Definition 1. Let f be a function defined on an interval I. Let $x_1, x_2 \in I$ with $x_1 < x_2$. The *chord* from $(x_1, f(x_1))$ to $(x_2, f(x_2))$ is the function

$$k : [x_1, x_2] \to \mathbb{R} \quad \text{given by} \quad k(x) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1) + f(x_1).$$

We say that f is *concave up on I* if $k(x) \geq f(x)$ for all $x_1, x_2 \in I$.

We say that f is *concave down on I* if $k(x) \leq f(x)$ for all $x_1, x_2 \in I$.

Thus, the graph of k is the line segment from $(x_1, f(x_1))$ to $(x_2, f(x_2))$; f is concave up if this line segment always lies above the graph of f, and f is concave down if this line segment always lies below the graph of f.

Definition 2. Let f be defined in an interval containing c.

We say that f has a *point of inflection* at c if the graph of f has a tangent line at c, and the concavity of f changes at c.

Theorem 1. (Second Derivative Test for Concavity)
Let f be twice differentiable at c and suppose $f'(c) = 0$.

- *If $f''(c) > 0$, then f has a local minimum at c.*
- *If $f''(c) < 0$, then f has a local maximum at c.*

Theorem 2. (Second Derivative Test for Local Extrema)
Let f be twice differentiable on an open interval I.

- *If $f'' > 0$ on I, then f is concave up on I.*
- *If $f'' < 0$ on I, then f is concave down on I.*

Problem 1. Let

$$f(x) = x^2 - 4x + 3.$$

Find intervals on which f is concave up and concave down.

Problem 2. Let

$$f(x) = x^3 - 3x + 3.$$

Find intervals on which f is concave up and concave down.

Problem 3. Let

$$f(x) = x^4 - 2x^2.$$

Find the points of inflection of f.

Problem 4. Let

$$f(x) = \frac{1}{1 + x^2}.$$

Find the points of inflection of f.

Problem 5. Let

$$f(x) = \begin{cases} x - n & \text{if } x \in [n, n+1) \text{ for an even integer } n \\ n - x + 1 & \text{if } x \in [n, n+1) \text{ for an odd integer } n \end{cases}$$

Find intervals on which f is concave up and concave down.

Problem 6. Let
$$f : [0, 4\pi] \to \mathbb{R} \quad \text{be given by} \quad f(x) = \sin x.$$

(a) Find all critical points of f.

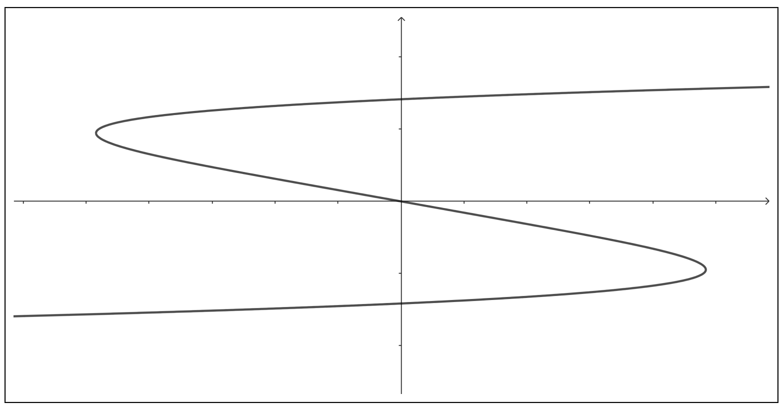
(b) Find all points of inflection of f.

(c) Find intervals on which f is increasing and decreasing.

(d) Find intervals on which f is concave up and concave down.

Problem 7. Find the indefinite integral
$$\int \left(\frac{\sec(1/x)}{x} \right)^2 dx.$$

Problem 8. Consider the function
$$f(x) = x^5 - 80x.$$
This function has three maximal branches of inverse, as indicated by the graph of $x = y^5 - 80y$, shown below.



Find the domain of each maximal branch of inverse of f.

Problem 9. Let R be the region in the region bounded by $y = (x-2)^2$ and $y = x$. Find the area of R.

Problem 10. Let R be the region in the region bounded by $y = (x-2)^2$ and $y = x$. Find the volume of the solid obtained by revolving R about the x-axis.